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Five Dimensional Axially Symmetric String Cosmological Models with Bulk Viscous Fluid

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Abstract Bulk viscous fluid distribution with massive strings in LRS Bianchi type-1 space time is studied. The exact solutions of the field equations are obtained by using the equation of state $\rho = -\lambda$ and $\rho = \lambda$. We observed that the bulk viscous fluid does not survive for $\rho = -\lambda$ whereas it survives for $\rho = \lambda$. Some physical and geometrical properties of the models are discussed.

Keywords Five dimensions · Bulk viscous fluid · String cosmology · General theory of relativity

1 Introduction

The study of bulk viscous mechanism in cosmology attracted the attention of many workers due to its significant role in the description of high entropy of the present universe [39, 40]. The material distribution behaves like a viscous fluid during early phase of the evolution of the universe when galaxies were formed [7]. Bulk viscous fluid may also arise due to the decay of massive superstring modes into mass less modes [33], gravitational string production [5, 38] and particle creation effects in the grand unification era [41]. It is well known that in several circumstances during cosmic evolution in which viscosity could arise [8, 12, 24] and lead to an effective mechanism of entropy production. Murphy [32] constructed isotropic homogeneous spatially flat cosmological models with bulk viscous fluid alone because the shear viscosity can not exist due to the assumption of isotropy and showed that the big

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bang singularity can be avoided by the introduction of bulk viscosity. Mohanty and Pradhan [28] constructed the problem of interactions of a gravitational field with bulk viscous fluid in Robertson Walker space-time. Mohanty and Pradhan [29] constructed Robertson Walker Cosmological model with bulk viscous fluid and equation of state $p = (\gamma - 1)\rho$ where $0 \le \gamma \le 2$. Mohanty and Pattanaik [27] investigated the anisotropic cosmological models with constant bulk viscous coefficient. Mohanty et al. [31] investigated an inflationary higher dimensional string cosmological model with constant bulk viscous coefficient in Lyra manifold and showed that the model does not admit initial singularity. Bali and Singh [3] constructed a string cosmological model with constant bulk viscous coefficient in General theory of relativity and showed that the model represents shearing and non rotating universe. Further Bali and Singh [4] revealed that the existence of Bianchi type-V cosmological model depends on the relationship of coefficient of bulk viscosity and expansion (θ). Rahaman et al. [34] constructed a Kaluza-Klein cosmological model with bulk viscous fluid considering time dependent equation of state $p = \lambda(t)\rho$. Recently Mohanty and Samanta [30] constructed various five dimensional physically realistic cosmological models with bulk viscous fluid in General theory of relativity.

Our universe is embedded in a higher dimensional space time, in view of Kaluza-Klein theory [17]. Earlier such cosmological models were investigated by Forgacs and Horvath [9] and Chodos and Detweiler [6]. For these models many important results were obtained using the idea that the size of extra dimension is small, as needed for the real universe [2, 10, 20, 21, 35, 37]. The cosmological dimensional reduction process is based on the idea that at very early stage all dimensions of the universe are comparable. Later, the scale of the extra dimensions became very small as to be unobservable by experiencing contraction. This idea was proposed by Chodos and Detweiler [6] who showed that, in the framework of a pure gravitational theory of Kaluza-Klein the extra dimension contracts to a very small size, while other three spatial dimensions expand isotropically. Guth [11] and Alvarez and Gavela [1] observed that during contraction process extra dimensions produce large amount of entropy. In recent years there has been a lot of interest in string cosmological models of the universe as they are believed to give rise to density perturbations leading to the formation of galaxies [42]. The existence of large scale of strings in the early universe does not contradict with present day observations of the universe. Cosmic strings may be created during phase transitions in the early era [13] and they act as a source of gravitational field. Letelier [19], Krori et al. [14, 15], Maharaj and Beesham [23], Reddy [36], Mahanta and Mukharjee [22], Mohanty and Mahanta [25, 26], Mohanty et al. [31] are some of the authors who have studied various aspects of string cosmologies in general relativistic theory as well as in alternative theories of gravitation. Very recently Mohanty and Samanta [30] constructed five dimensional string cosmological models with bulk viscosity in general theory of relativity. In this paper we constructed five dimensional LRS Bianchi type-1 string cosmological models with bulk viscosity in general relativity.

2 The Metric and the Field Equations

Here we consider the five dimensional LRS Bianchi type-1 metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(dy^2 + dz^2) + C dm^2 \tag{1}$$

where A, B and C are functions of cosmic time 't' only.

We assume that the co ordinates to be commoving so that

$$u^0 = 1, \qquad u^1 = u^2 = u^3 = u^4 = 0 \tag{2}$$

The energy momentum tensor for a cloud of string dust with a bulk viscous fluid of the string is given by Letelier [18] and Landau and Lifshitz [16] in the form of

$$T_{ij} = \rho u_i u_j - \lambda w_i w_j - \xi u^l_{;l}(g_{ij} + u_i u_j) \tag{3}$$

where ξ is the bulk viscous coefficient ρ the proper energy density for a cloud of strings with particle attached to them, λ the string tension density, the expansion $\theta = u^l_{;l}$, u^i the five velocities of the particles, g_{ij} the fundamental tensor, w^i the unit space like vector representing the direction of the string satisfying

$$u^i u_i = -w^i w_i = -1 \tag{4}$$

and

$$u^i w_i = 0 \tag{5}$$

The Einstein field equations for a system of strings are given by Letelier [19] as

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij} \tag{6}$$

Using (2)–(5) the explicit forms of the field equation (6) for the metric (1) are obtained as

$$-\left(\frac{\dot{B}}{B}\right)^2 - 2\frac{\dot{A}\dot{B}}{AB} - 2\frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} = -\rho \tag{7}$$

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{B}\dot{C}}{BC} = \lambda + \xi\theta \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \xi\theta \tag{9}$$

and

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = \xi\theta \tag{10}$$

Here afterwards the dot represents ordinary differentiation with respect to time 't'. In order to derive some admissible exact solutions of the field equations (7)–(10), we use the following scale transformations

$$A = e^{\alpha}, \quad B = e^{\beta} \quad \text{and} \quad C = e^{\gamma} \tag{11}$$

$$dt = AB^2 C\, dT \tag{12}$$

The field equations (7)–(10) reduce to

$$-\beta'^2 - 2\alpha'\beta' - 2\beta'\gamma' - \alpha'\gamma' = -\rho e^{2\alpha+4\beta+2\gamma} \tag{13}$$

$$2\beta'' + \gamma'' - \beta'^2 - 2\alpha'\beta' - 2\beta'\gamma' - \alpha'\gamma' = (\lambda + \xi\theta) e^{2\alpha+4\beta+2\gamma} \tag{14}$$

$$\alpha'' + \beta'' + \gamma'' - 2\alpha'\beta' - 2\beta'\gamma' - \alpha'\gamma' - \beta'^2 = \xi\theta e^{2\alpha+4\beta+2\gamma} \tag{15}$$

and

$$\alpha'' + 2\beta'' - 2\alpha'\beta' - 2\beta'\gamma' - \alpha'\gamma' - \beta'^2 = \xi\theta e^{2\alpha+4\beta+2\gamma} \tag{16}$$

Here afterwards the prime stands for $\frac{d}{dT}$.

From field equations (15) and (16) we obtain

$$\beta = \gamma + a_1 T + a_2 \tag{17}$$

where a_1 and a_2 are constants of integration.

Using (17), field equations (13)–(16) are solvable for any arbitrary function γ. For the sake of simplicity here we consider

$$\gamma = kT + k_1 \tag{18}$$

where $k \neq 0$ and k_1 are arbitrary constants. Using (18) in (17), we get

$$\beta = k_2 T + k_3 \tag{19}$$

where $k_2 \neq 0$ and k_3 are arbitrary constants. Now using (18) and (19) in the field equations (13)–(16), we obtained

$$-k_2^2 - 2\alpha' k_2 - 2kk_2 - \alpha' k = -\rho e^{2\alpha+4\beta+2\gamma} \tag{20}$$

$$-k_2^2 - 2\alpha' k_2 - 2kk_2 - \alpha' k = (\lambda + \xi\theta) e^{2\alpha+4\beta+2\gamma} \tag{21}$$

and

$$\alpha'' - k_2^2 - 2\alpha' k_2 - 2kk_2 - \alpha' k = \xi\theta e^{2\alpha+4\beta+2\gamma} \tag{22}$$

In order to avoid the mathematical complexity due to highly nonlinear nature of the field equations, we consider following physical conditions:

$$\rho + \lambda = 0 \tag{23}$$

(Mohanty and Samanta [30]; Mohanty et al. [32]) and

$$\rho = \lambda \tag{24}$$

(geometric string [16, 19])

2.1 Case 1: $\rho + \lambda = 0$

In this case using (23) in the field equations (20)–(22), we get

$$\alpha = k_4 + k_5 e^{(2k_2+k)T} - \frac{2kk_2 + k_2^2}{2k_2 + k} T \tag{25}$$

and

$$\xi = 0 \tag{26}$$

where k_4 and k_5 are arbitrary constants.

Hence the geometry of the model is described by the metric

$$ds^2 = -\text{Exp}\left(2k_4 + 2k_5 e^{(2k_2+k)T} - \frac{4kk_2 + 2k_2^2}{2k_2 + k}T + 4k_2 T + 2kT + 4k_3 + 2k_1\right)dT^2 + \text{Exp}\left(2k_4 + 2k_5 e^{(2k_2+k)T} - \frac{4kk_2 + 2k_2^2}{2k_2 + k}T\right)dX^2 + \text{Exp}(2k_2 T + 2k_3)(dY^2 + dZ^2) + \text{Exp}(2kT + 2k_1)dM^2 \tag{27}$$

The rest energy density (ρ) and string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ), the spatial volume (V) and the deceleration parameter (q) for the model (27) are obtained as

$$\rho(= -\lambda) = \frac{(k + 2k_2)^2 k_5 e^{(k+2k_2)T}}{\text{Exp}(2k_4 + 2k_5 e^{(2k_2+k)T} - \frac{4kk_2+2k_2^2}{2k_2+k}T + 4k_2 T + 2kT + 4k_3 + 2k_1)} \tag{28}$$

$$\rho_p = \frac{2(k + 2k_2)^2 k_5 e^{(k+2k_2)T}}{\text{Exp}(2k_4 + 2k_5 e^{(2k_2+k)T} - \frac{4kk_2+2k_2^2}{2k_2+k}T + 4k_2 T + 2kT + 4k_3 + 2k_1)} \tag{29}$$

$$\theta = \frac{k_5(2k_2 + k)e^{(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k} + 2k_2 + k}{\text{Exp}(k_4 + k_5 e^{(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k}T + 2k_2 T + kT + 2k_3 + k_1)} \tag{30}$$

$$\sigma^2 = \frac{1}{2}\Bigg[\left(\frac{(2k_2 + k)k_5 e^{(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k}}{\text{Exp}(k_4 + k_5 e^{(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k}T)}\right)^2 + \frac{1}{4} + \frac{(2k_2 + k)k_5 e^{(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k}}{\text{Exp}(k_4 + k_5 e^{(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k}T)} + 2\left(\frac{k_2^2}{(k_2 T + k_3)^2} + \frac{1}{4} + \frac{k_2}{k_2 T + k_3}\right) + \left(\frac{k}{kT + k_1}\right)^2 + \frac{1}{4} + \frac{k}{kT + k_1}\Bigg] \tag{31}$$

$$V = \text{Exp}\left(k_4 + k_5 e^{(2k_2+k)T} - \frac{2kk_2 + k_2^2}{2k_2 + k}T + 2k_2 T + kT + 2k_3 + k_1\right) \tag{32}$$

and

$$q = -\left[1 + \frac{(2k_2 + k)^2 k_5 e^{(2k_2+k)T}}{((2k_2 + k)k_5 e^{(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k} + 2k_2 + k)^2}\right] \tag{33}$$

The reality condition $\rho > 0$ is satisfied when $k_5 > 0$. At early era ($T \to 0$), we have $\rho > 0$, $\rho_p > 0$. Also we get

$$\frac{\rho_p}{|\lambda|} = 2 \tag{34}$$

This indicates that strings and particles coexist. From (34) we observe that the particles dominate over the strings. Moreover, we observe that the three spatial co-ordinates expand indefinitely for $k_2 > 0$ but the extra dimension contracts as $T \to \infty$ for $k < 0$. For $k_5 = 0$, $k < 0$ and $k_2 < 0$, we observe that string does not survive and (27) represents a contracting model. For $k_5 > 0$, $k < 0$, $k_2 > 0$ and $k + 2k_2 \neq 0$ (27) represents a physically realistic string cosmological model. Further $\lim_{T\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$. Therefore the model does not approach isotropy for large value of T. The spatial volume (V) is finite when $T = 0$ and it becomes infinity when $T \to \infty$. From (26) we obtained that, bulk viscous fluid does not survive for the equation of state ($\rho = -\lambda$). Equation (33) represents negative deceleration parameter. This indicates the existence of inflationary cosmological model.

2.2 Case 2: $\rho = \lambda$

In this case using (24) in field equations (20)–(22) we get

$$\alpha = k_6 + k_7 e^{-(2k_2+k)T} - \frac{2kk_2 + k_2^2}{2k_2 + k} T \tag{35}$$

and

$$\xi = \frac{2(k_2+k)^2 k_7 e^{-(2k_2+k)T}}{(-(2k_2+k)k_7 e^{-(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k}T + 2k_2 + k)\mathrm{Exp}(k_6 + k_7 e^{-(2k_2+k)T} - \frac{2kk_2+k^2}{2k_2+k}T + 2k_2 T + 2k_3 + kT + k_1)} \tag{36}$$

where k_7 and k_6 are arbitrary constants of integration. The geometry of the model described the metric

$$\begin{aligned} ds^2 = &-\mathrm{Exp}\left(2k_6 + 2k_7 e^{-(2k_2+k)T} - \frac{2(2kk_2 + k_2^2)}{2k_2+k}T + 4k_2 T + 4k_3 + 2kT + 2k_1\right) dT^2 \\ &+ \mathrm{Exp}\left(2k_6 + 2k_7 e^{-(2k_2+k)T} - \frac{2kk_2 + k_2^2}{2k_2+k}T\right) dX^2 \\ &+ \mathrm{Exp}(2k_2 T + 2k_1)(dX^2 + dY^2) + \mathrm{Exp}(2kT + 2k_1) dm^2 \end{aligned} \tag{37}$$

In this case physical and kinematical quantities are obtained as

$$\rho(=\lambda) = -\frac{(2k_2+k)^2 k_7 e^{-(2k_2+k)T}}{\mathrm{Exp}(2k_6 + 2k_7 e^{-(2k_2+k)T} - \frac{2(2kk_2+k_2^2)}{2k_2+k}T + 4k_2 T + 4k_3 + 2kT + 2k_1)} \tag{38}$$

$$\theta = \frac{-(2k_2+k)k_7 e^{-(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k}T + 2k_2 + k}{\mathrm{Exp}(k_6 + k_7 e^{-(2k_2+k)T} - \frac{(2kk_2+k_2^2)}{2k_2+k}T + 2k_2 T + 2k_3 + kT + k_1)} \tag{39}$$

$$\begin{aligned} \sigma^2 = \frac{1}{2}\Bigg[&\frac{(-(2k_2+k)k_7 e^{-(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k})^2 + 2k_2^2 + k^2}{\mathrm{Exp}(2k_6 + 2k_7 e^{-(2k_2+k)T} - \frac{2(2kk_2+k_2^2)}{2k_2+k}T + 4k_2 T + 4k_3 + 2kT + 2k_1)} \\ &+ \frac{-(2k_2+k)k_7 e^{-(2k_2+k)T} - \frac{2kk_2+k_2^2}{2k_2+k}T + 2k_2 + k}{\mathrm{Exp}(k_6 + k_7 e^{-(2k_2+k)T} - \frac{(2kk_2+k_2^2)}{2k_2+k}T + 2k_2 T + 2k_3 + kT + k_1)} + 1\Bigg] \end{aligned} \tag{40}$$

$$V = \mathrm{Exp}\left(k_6 + k_7 e^{-(2k_2+k)T} - \frac{(2kk_2 + 3k_2^2 + k^2)}{2k_2+k}T + 2k_3 + k_1\right) \tag{41}$$

and

$$q = -\left[-1 + \frac{(2k_2+k)^2 k_7 e^{-(2k_2+k)T}}{(-(2k_2+k)k_7 e^{-(2k_2+k)T} + \frac{2kk_2+3k_2^2+k^2}{2k_2+k})^2}\right] \tag{42}$$

From (38) we observed that $\rho \to 0$ as $T \to \infty$ and satisfy the reality condition for $k_7 < 0$. Moreover we observe that the scale factors A and B expand indefinitely for $k_2 > 0$ whereas C contracts as $T \to \infty$ for $k < 0$. For $k_7 = 0$, $k < 0$ and $k_2 < 0$, we observe that string and bulk viscous fluid do not survive and (37) represents a contracting model. For $k_7 < 0$, $k < 0$, $k_2 > 0$ and $2k_2 + k \neq 0$, we obtained a physically realistic bulk viscous string cosmological model. Further $\lim_{T\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy for large value of T. The spatial volume V is finite for $T = 0$ and it becomes infinite for large value of T. In this case we obtained negative variable parameter; this yields an inflationary cosmological model.

3 Conclusion

In this paper we constructed two five dimensional string cosmological models with bulk viscous fluid in General theory of relativity. We observe that the bulk viscosity does not survive for the equation of state $\rho = -\lambda$, whereas bulk viscosity survives for the equation of state $\rho = \lambda$ (Geometric string). Further we observed that both the models are free from initial singularity, this supports the analysis of Murphy [32] i.e. the introduction of bulk viscous fluid avoids the initial singularity.

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